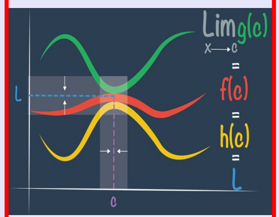
Calculus I

Lecture 37



Feb 19-8:47 AM

Class QZ 18

Verify that $f(x) = \frac{1}{x}$ Satisfies the conditions of MVT over $[\frac{1}{4}, 1]$, then find all numbers c that Satisfy the conclusion of MVT.

$f(x) \to$ Domain $\to x \neq 0$

$f(1) = 1$, $f(\frac{1}{4}) = 4$

$f(x)$ is Cont. on $[\frac{1}{4}, 1]$

$f'(x) = \frac{-1}{x^2}$

$f'(x)$ is diff. on $(\frac{1}{4}, 1)$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$\frac{-1}{c^2} = \frac{1-4}{1-\frac{1}{4}}$ $\quad \frac{-1}{c^2} = \frac{-3}{\frac{3}{4}} \to \frac{-1}{c^2} = 4$

$-4c^2 = -1 \to c^2 = \frac{1}{4}$ $\quad c = \pm\sqrt{\frac{1}{4}}$ $\quad c = \pm\frac{1}{2}$

$\boxed{c = \frac{1}{2} \text{ is in } (\frac{1}{4}, 1)}$

Apr 17-9:35 AM

$f(x)$ is an odd function. $\Rightarrow$ Symmetric w/t the origin.

$f'(x) < 0$ for $0 < x < 2$

$f'(x) > 0$ for $x > 2$

$f''(x) > 0$ for $0 < x < 3$

$f''(x) < 0$ for $x > 3$

$\lim_{x \to \infty} f(x) = -2$

Draw its graph

an odd function always contains the origin.

$f(0) = -f(-0)$

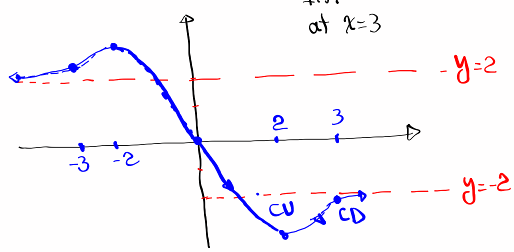
$f(0) = -f(0)$

$2f(0) = 0$ $\rightarrow f(0) = 0$

x	0	2	3	∞
$f'(x)$	$-$	$+$	$+$	
$f''(x)$	$+$	$+$	$-$	
$f(x)$	Dec., CU	Inc., CU	Inc., CD	

I.P. at $x = 3$

Since $f'(x)$ is defined, $f(x)$ is diff., which implies Cont.



Apr 18-8:58 AM

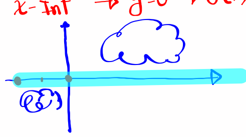
$f(x) = x\sqrt{x+2}$

Domain $\rightarrow x+2 \geq 0 \rightarrow x \geq -2 \rightarrow$ Domain $[-2, \infty)$

Y-Int $\rightarrow x = 0 \rightarrow f(0) = 0\sqrt{0+2} = 0 \rightarrow$ Y-Int $(0,0)$

x-Int $\rightarrow y = 0 \rightarrow f(x) = 0 \rightarrow x\sqrt{x+2} = 0 \rightarrow x = 0,\ x = -2$

$\rightarrow$ x-Ints $(0,0)$ & $(-2,0)$



$-2 < x < 0 \rightarrow f(x) < 0$ Below x-axis

$x > 0 \rightarrow f(x) > 0$ Above x-axis

$f(x) = x\sqrt{x+2}$

$= \sqrt{x^2(x+2)} = \sqrt{x^3+2x^2} = (x^3+2x^2)^{1/2}$

$f'(x) = \frac{1}{2}(x^3+2x^2)^{-1/2} \cdot (3x^2+4x) = \frac{x(3x+4)}{2\sqrt{x^3+2x^2}} = \frac{x(3x+4)}{2x\sqrt{x+2}}$

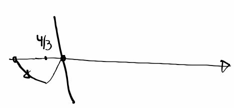
$f'(x) = \frac{3x+4}{-2\sqrt{x+2}}$ for $x < 0$ $\quad \sqrt{x^2} = -x$

$f'(x) = \frac{3x+4}{2\sqrt{x+2}}$ for $x > 0$ $\quad \sqrt{x^2} = x$

$f'(x) = 0 \rightarrow 3x+4 = 0 \rightarrow x = \frac{-4}{3} = -1\frac{1}{3}$

$f'(x)$ is undefined $\rightarrow \sqrt{x+2} = 0 \rightarrow x = -2$

x	-2	-1.5	$-\frac{4}{3}$	∞
$f'(x)$	$-$		$+$	
$f(x)$	Dec.		Inc.	



Apr 18-9:10 AM

$f(x) = x\sqrt{x+2}$ $\quad f(x) = x(x+2)^{1/2}$

$f'(x) = 1\cdot(x+2)^{1/2} + x\cdot\frac{1}{2}(x+2)^{-1/2}\cdot 1$

$= \frac{2}{2}(x+2)^{1/2} + \frac{1}{2}x(x+2)^{-1/2}$

$= \frac{1}{2}(x+2)^{-1/2}\left[2(x+2)^{1} + x\right] = \frac{1}{2}(x+2)^{-1/2}(3x+4)$

$f'(x) = \dfrac{3x+4}{2\sqrt{x+2}} \longrightarrow (x+2)^{1/2}$

$f''(x) = \frac{1}{2}\cdot\dfrac{3\sqrt{x+2} - (3x+4)\cdot\frac{1}{2}(x+2)^{-1/2}\cdot 1}{(\sqrt{x+2})^2}$

$= \frac{1}{2}\ \dfrac{3\sqrt{x+2} - \frac{3x+4}{2\sqrt{x+2}}}{x+2}$ Multiply by $2\sqrt{x+2}$

$= \frac{1}{2}\cdot\dfrac{2\sqrt{x+2}\left(3\sqrt{x+2} - \frac{3x+4}{2\sqrt{x+2}}\right)}{2\sqrt{x+2}\,(x+2)}$

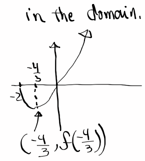
$= \frac{1}{2}\cdot\dfrac{6(x+2) - (3x+4)}{2(x+2)\sqrt{x+2}} = \boxed{\dfrac{3x+8}{4(x+2)\sqrt{x+2}}}$

$f''(x) = 0 \rightarrow 3x+8=0 \rightarrow x = -\frac{8}{3} = -2.\overline{6}$ Not usefull since it is not in the domain.

$f''(x)$ is undefined at $x=-2$

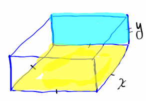
x	-2	$-4/3$	∞
$f'(x)$	$-$	•	$+$
$f''(x)$	$+$		$+$
$f(x)$	Dec., CU		Inc., CU

NO I.P.



Apr 18-9:26 AM

1200 cm^2 of materials are available to make a box with square base & open top.



we need materials
for base x^2
for sides $4xy$

Find dimensions to make largest Volume.

$x^2 + 4xy = 1200 \longrightarrow 4xy = 1200 - x^2$

$y = \dfrac{1200 - x^2}{4x}$

$V = x^2 y$

$V(x) = x^{\cancel{2}}\left[\dfrac{1200 - x^2}{4\cancel{x}}\right]$

$V(x) = \dfrac{x(1200 - x^2)}{4}$ $\quad V(x) = \frac{1}{4}\left[1200x - x^3\right]$

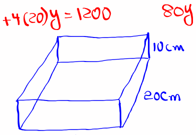
$V'(x) = \frac{1}{4}\left[1200 - 3x^2\right] = \frac{3}{4}\left[400 - x^2\right]$

$x = \pm 20 \rightarrow \boxed{x=20}$

$V''(x) = \frac{3}{4}\left[-2x\right]$ $\quad V''(20) < 0 \longrightarrow$ C.D. Max

$x^2 + 4xy = 1200$ $\quad 80y = 1200 - 400$

$20^2 + 4(20)y = 1200$ $\quad 80y = 800 \rightarrow \boxed{y=10}$



Box is 20×20×10 cm.
Largest Volume 4000 cm^3

Google
1) MVT Proof
2) First - Derivative Test
3) Second - Derivative Test

Continue working on SG.

Apr 18-9:41 AM